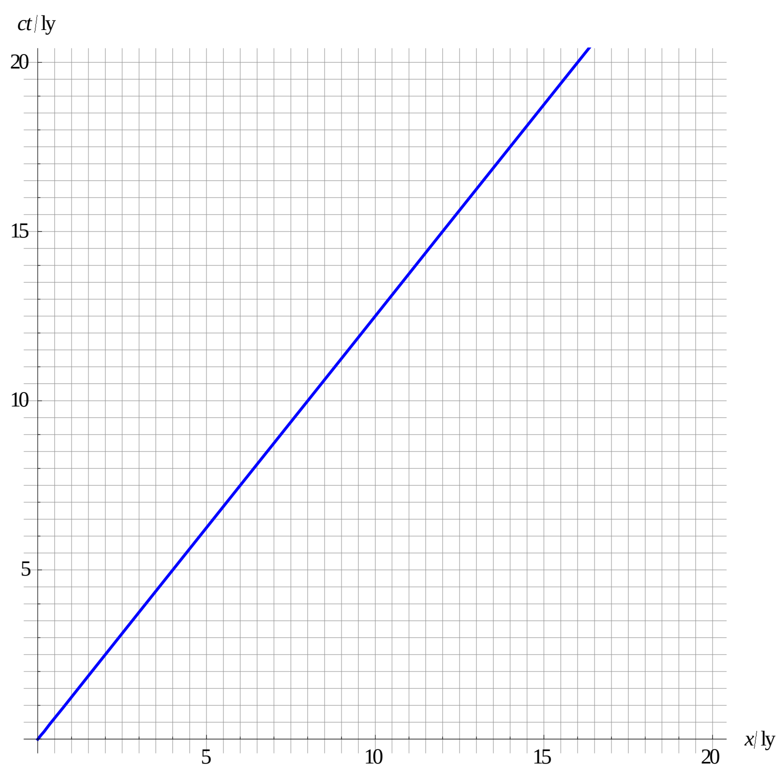
Problem for week 46

Special Relativity

(a) Describe an experiment that supports special relativity.
(b) State the postulates of relativity.
(c) A rocket X moves past earth with velocity +0.60c. A second rocket Y moves past earth with velocity –0.80c. Determine the velocity of Y relative to X according to
(i) Galilean relativity,
(ii) Einstein relativity.

(d) A rocket leaves earth with speed 0.80c towards a space station at rest relative to earth, 16 ly away according to earth. Calculate the time the rocket and space station meet according to
(i) earth clocks,
(ii) rocket clocks.
(e) When 3.0 years have gone by according to the rocket, the rocket sends a radio signal back to earth. Determine the coordinates of the emission event in the earth frame.
(f) At the instant the rocket left earth, a probe left the space station towards earth with speed 0.20c. Determine when the rocket and the probe meet according to earth.
(g) The spacetime diagram shows the worldline of the rocket.



(i) Draw dots separated by 1 year according to rocket clocks on this worldline.
(ii) Use the diagram to verify the answers to (c) (i) and (c) (ii).

(iii) On the same diagram draw the worldline of the probe.

(iv) Use the diagram to predict when the rocket meets the probe according to rocket clocks.

(v) Verify algebraically the answer to (iv).

Answers

(a) Muon decay - See textbook.

(b) The speed of light is the same for all inertial observers and all the laws of physics are the same in all inertial frames.

(c)

(i) $-1.4c$

(ii) $\frac{-0.60c-0.80c}{1-0.60c(-0.80c)}=-0.946c\approx-0.95c$

(d)

(i) $\frac{16\text{ ly}}{0.80c}=20\text{ yr}$.

(ii) For the rocket, leaving earth and arriving at space station occur at the same pint in space so they measure a proper time interval for these 2 events. Hence $20=\gamma\tau\Rightarrow\tau=\frac{20}{\gamma}=\frac{20}{\frac{5}{3}}=12\text{ yr}$.

OR

distance separating earth and space station is $\frac{16}{\gamma}=\frac{16}{\frac{5}{3}}=\frac{48}{5}\Rightarrow\tau=\frac{\frac{48}{5}}{0.80c}=12\text{ yr}$.

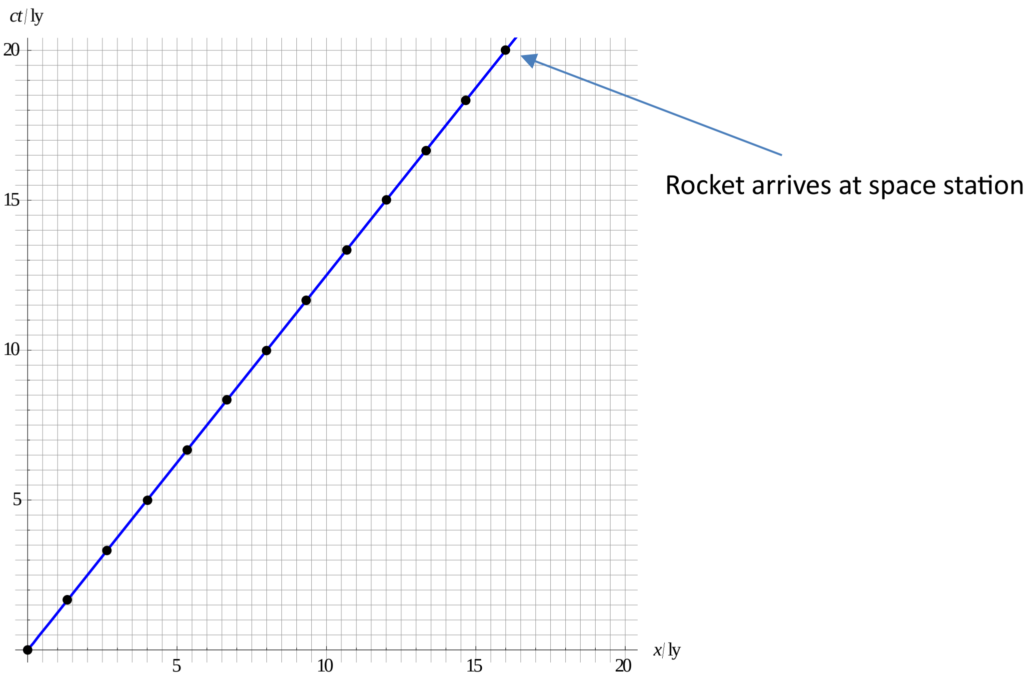
(e) In the rocket frame the event has coordinates $x'=0$, $ct'=3.0\text{ ly}$. Hence,

$x=\gamma(x'+vt')=\frac{5}{3}\times(0+0.80c\times3.0)=4.0\text{ ly}$ and $ct=\gamma(ct'+\frac{v}{c}x')=\frac{5}{3}\times(3.0+0)=5.0\text{ ly}$.

(f) The distance between the rocket and the probe is decreasing at a rate of c so the time taken is 16 years.

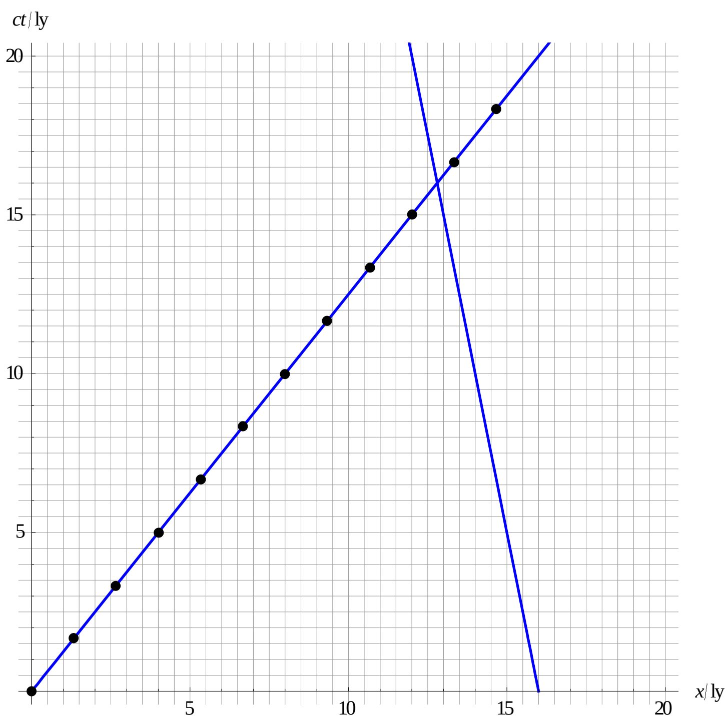
(g)

(i)



(ii) The rocket reaches the space station when x = 16 ly, i.e. at 20 years for earth. For the rocket it is 12 years.

(iii)



(iv) At about 9.6 years.

(v) The rocket meets the probe in 16 years according to earth. The rocket is then a distance of 0.80c×16=12.8 ly away. Hence $ct' = \gamma(ct - \frac{v}{c}x) = \frac{5}{3} \times (16 - 0.80 \times 12.8) = 9.6 \text{ ly}$, hence 9.6 years.